

**LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034**

B.Sc. DEGREE EXAMINATION – MATHEMATICS

FIFTH SEMESTER – November 2009

**MT 5507 - OPERATIONS RESEARCH**

Date & Time: 7/11/2009 / 9:00 - 12:00

Dept. No.

Max. : 100 Marks

**SECTION – A**

**Answer ALL the questions:**

**(10 x 2 = 20)**

1. Give the standard form of a linear programming problem.
2. When does a LP problem have a degenerate solution?
3. How do we deal with degeneracy in a transportation problem?
4. How do we convert a maximization assignment problem to a minimization problem?
5. Define saddle point of a two person zero sum game.

6. Find the minimax of the following matrix  $\begin{pmatrix} 1 & 3 & 6 \\ 2 & 1 & 3 \\ 6 & 2 & 1 \end{pmatrix}$

7. Define earliest start time and earliest finish time of a project.
8. What is slack of an activity in a project?
9. Define EOQ.
10. What are the different costs associated with holding an inventory? Explain.

**SECTION – B**

**Answer any FIVE questions:**

**(5 X 8 = 40)**

11. Solve using Graphical method:

Minimise  $z = 7x_1 + 8x_2$  subject to

$$3x_1 + x_2 \geq 8, \quad x_1 + 3x_2 \geq 11, \quad x_1, x_2 \geq 0$$

12. Solve the following assignment problem:

	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	R <sub>4</sub>	R <sub>5</sub>
C <sub>1</sub>	11	17	8	16	20
C <sub>2</sub>	9	7	12	6	15
C <sub>3</sub>	13	16	15	12	16
C <sub>4</sub>	21	24	17	28	26
C <sub>5</sub>	14	10	12	11	15

13. For the game with pay – off matrix :

	Player B	
Player A	B <sub>1</sub>	B <sub>2</sub>
A <sub>1</sub>	6	-3
A <sub>2</sub>	-3	0

Determine the optimal strategies for players A and B. Also determine the value of the game.

14. Obtain an optimum basic feasible solution to the following transportation problem:

		Destination				
		D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	Factory capacity
Source	S <sub>1</sub>	19	30	50	10	7
	S <sub>2</sub>	70	30	40	60	9
	S <sub>3</sub>	40	8	70	20	18
	Demand	5	8	7	14	

15. Construct the network diagram comprising activities B, C, ..., Q and N such that the following constraints are satisfied:

$$B < E, F; \quad C < G, L; \quad E, G < H; \quad F, G < H; \quad L, H < I;$$

$L < M; \quad H, M < N; \quad H < J; \quad I, J < P; \quad P < Q$ . The notation  $X < Y$  means that an activity X must be finished before Y can begin. ( Indicate the dummy activities as dotted lines ).

16. Use simplex method to solve the following LP problem:

Maximise  $Z = 2x_1 + 4x_2 + x_3 + x_4$  subject to the constraints

$$x_1 + 3x_2 + x_4 \leq 4, \quad 2x_1 + x_2 \leq 3, \quad x_2 + 4x_3 + x_4 \leq 3, \quad x_1, x_2, x_3, x_4 \geq 0.$$

17. Explain the method of solving an assignment problem.

18. A company plans to consume 760 pieces of a particular component. Past records indicate that purchasing department spent Rs. 12, 555 for placing 15, 500 purchase orders. The average inventory was valued at Rs. 45, 000 and the total storage cost was Rs. 7, 650 which included wages, rent, taxes, insurance etc. related to store department. The company borrows capital at the rate of 10% a year. If the price of a component is Rs. 12 and lot size is 10, find the following: (i) purchase price per year, (ii) purchase expenses per year, (iii) Storage expenses per year, (iv) capital cost per year, (v) total cost per year.

### SECTION – C

Answer any TWO questions:

(2 X 20 = 40)

19. Use the Big – M method to solve the following LP problem:

Maximise  $Z = 6x_1 + 4x_2$  subject to the constraints

$$2x_1 + 3x_2 \leq 30, \quad 3x_1 + 2x_2 \leq 24, \quad x_1 + x_2 \geq 3, \quad x_1, x_2 \geq 0. \text{ Give the alternate solution.}$$

20. Solve the following transportation problem:

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	A <sub>i</sub>
O <sub>1</sub>	4	7	3	8	2	4
O <sub>2</sub>	1	4	7	3	8	7
O <sub>3</sub>	7	2	4	7	7	9
O <sub>4</sub>	4	8	2	4	7	2
b <sub>j</sub>	8	3	7	2	2	

Where A<sub>i</sub> is availability at origin O<sub>i</sub> and b<sub>j</sub> is the requirement at destination D<sub>j</sub> and cell entries are unit costs of transportation from any origin to any destination.

21. (a) Solve the game whose payoff matrix is given below by graphical method:

	Player B			
Player A	B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>
A <sub>1</sub>	4	-2	3	1
A <sub>2</sub>	-1	2	0	1
A <sub>3</sub>	-2	1	-2	0

(b)The following table lists the jobs of a network along with their time estimates:

Job		Duration (Days)		
i	j	Optimistic	Most likely	Pessimistic
1	2	3	6	15
1	6	2	5	14
2	3	6	12	30
2	4	2	5	8
3	5	5	11	17
4	5	3	6	15
6	7	3	9	27
5	8	1	4	7
7	8	4	19	28

Draw the network diagram. Calculate the length and variance of the critical path after estimating the earliest and latest event times for all nodes. Find the probability of completing the project before 31 days. What is the chance of project duration exceeding 46 days?

22. (a) Explain Dijkstra’s algorithm to determine the shortest route between the source node and every other node in a network.

(b) A contractor has to supply 20,000 units per day. He can produce 30,000 units per day. The cost of holding a unit in stock is Rs. 3 per year and the set – up cost per run is Rs. 50. How frequently and of what size, the production runs be made?